ELECTROMAGNETIC SCATTERING FROM TWO DIELECTRIC SPHERES:

COMPARISON BETWEEN THEORY AND EXPERIMENT

by

George W. Kattawar and Cleon E. Dean

Report No. 17

Department of Physics
Texas A&M University
College Station, Texas 77843

August 30, 1982

This report has been submitted to Optics Letters for publication.

REPORT DOCUMENTATION PAGE

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
Report No. 17	AD-A229048	

4. TITLE (and Subtitle)
Electromagnetic Scattering from Two Dielectric Spheres: Comparison Between Theory and Experiment

5. TYPE OF REPORT & PERIOD COVERED
Reprint of publication

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
George W. Kattawar and Cleon E. Dean

8. CONTRACT OR GRANT NUMBER(s)
ONR Contract
N00014-80-C-0113

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Department of Physics
Texas A&M University
College Station, Texas 77843

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS
Office of Naval Research
Navy Department, Code 480
Arlington, VA 22217

12. REPORT DATE
August 30, 1982

13. NUMBER OF PAGES
12

14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)

15. SECURITY CLASS. (of this report)
unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

The distribution of this report is unlimited.

DISTRIBUTION STATEMENT A

Approved for public release;
Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

electromagnetic scattering, two sphere scattering

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A comparison is made between theoretical and experimental results for cooperative scattering between two spheres. The overall agreement between theory and experiment is quite good. Also a large side scattering resonance which was measured to be 44 times larger than that due to a single sphere was calculated to be actually 47.6 times larger.

DD FORM 1473 EDITION OF 1 NOV 65 IS OBSOLETE
1 JAN 73
S/N 0102-014-6601 |

Electromagnetic Scattering from Two Dielectric Spheres:

Comparison Between Theory and Experiment

George W. Kattawar and Cleon E. Dean

Department of Physics

Texas A&M University

College Station, Texas 77843

ABSTRACT

A comparison is made between theoretical and experimental results
for cooperative scattering between two spheres. The overall agreement
between theory and experiment is quite good. Also a large side scatter-
ing resonance which was measured to be 44 times larger than that due to
a single sphere was calculated to be actually 47.6 times larger.

In a recent publication, Wang et al.[1] presented some extremely interesting measurements from the cooperative scattering from two spheres using microwave analog techniques. The results they presented in the above reference were only a small part of a more comprehensive study[2]. One of the more interesting discoveries in this study was the enormous enhancement of the scattered intensity component perpendicular to the scattering plane, at right angles to the incident beam for two identical spheres in contact when the axis of symmetry of the two spheres bisected the scattering angle.

It is the purpose of this short paper to demonstrate that their results are borne out beautifully by theoretical calculations. We have used the powerful technique presented by Bruning and Lo[3,4] to construct a very complex computer program to perform the calculations. It should be noted that several errors (probably typographical) were discovered in their equations and virtually all of them had to be rederived. In a future publication we will present a corrected set of equations along with several improvements for other researchers to use. The program was written in Fortran 77 employing complex, double precision arithmetic. The program was subjected to many rigorous tests. In Fig. 1 we show the results for extinction coefficient calculations as a function of distance of separation of centers ($kd=2\pi d/\lambda$). The case shown is for a size parameter x=0.9283 and refractive index N=1.54. If this case is compared to a similar case, calculated by Kattawar and Humphreys[5] (Fig. 6 of that reference) using the point dipole approximation, which is a completely independent method and program, it will be seen that the agreement is remarkably good.

We next calculated the following quantities to compare with the measurements of Schuerman and Wang[2]:

1) $|S(0)|$ - modulus of complex scattering amplitude in the forward direction

2) $P = \dfrac{4\pi}{k^2 G}\ Im[S(0)]$

3) $Q = \dfrac{4\pi}{k^2 G}\ Re[S(0)]$

4) $\phi(0)$ - phase angle defined as the angle between $S(0)$ and the P axis

where $G=2\pi a^2$ where a is the sphere radius and k is the wavenumber $2\pi/\lambda$. Table I gives the comparison between theory and experiment for three different size parameters and refractive indices when the spheres are in contact. Also for ease of visualization we have also presented the P versus Q plots in Figs. 2a-2c. There are several noteworthy features in the comparisons. First, the agreement between theory and experiment on the phase angles is within a few percent in all but one measurement ($x=3.752, \alpha=0°$) where it reaches 8%. Secondly, the worst disagreement in all parameters usually occurs at $\alpha=0°$, i.e., for end on incidence which is the region most difficult to measure. Finally, in the P-Q plots of Fig. 2, the theoretical results always converge closer to the non-interacting sphere limit (labelled NIS on the figures) for $\alpha=90°$ (broadside incidence) than the measurements. This fact is borne out more realistically in Fig. 2d where the spheres are separated by 3.71 diameters.

The truly exciting results are presented in Figs. 3a and 3b. In Fig. 3a we show the intensity component perpendicular to the scattering plane for a 90° scattering angle ($I_r(90°)$) as a function of particle orientation angle α. If this case is compared with the experimental re-

sults of Schuerman and Wang[2] (their Fig. 12A) the only major difference is the maximum value where they measure 37.5 and the calculations give 43.6. Also shown on this figure is the sensitivity of this peak to particle separation. When the spheres are separated by 1.5 sphere diameters (kd=14.034) the maximum almost becomes a minimum, decreasing more than a factor of ten. We also calculated the parallel component of the radiance when the electric vector is parallel to the scattering plane. It also shows a maximum at $\alpha=45°$ but it is roughly a factor of two smaller. To see if another maximum occurred in the orthogonal position we rotated the spheres in the opposite direction so that at $\alpha=45°$ the axis of symmetry of the spheres was perpendicular to the bisectrix. Another maximum occured but it was roughly five times smaller than the maximum which occurred when the axis of symmetry was in the specular position. In Fig. 3b we show a similar plot except the size is changed and the refractive index is now complex. This case can be compared with Fig. 2 of Wang et al[1]. Again the overall agreement is extremely good but as in the previous case we calculate a higher maximum, namely 50 compared to 46.2 or a factor of 47.6 times larger than the radiance for a single sphere. If the spheres are moved slightly apart (1.04 sphere diameters) the maximum falls to 44.5. If they are two diameters apart the maximum becomes a minimum but smaller secondary maxima start to appear. These results show that the combination of multiple scattering of the near fields of the two spheres with the direct scattered fields have just the right phase behavior to produce the enhancement.

The overall agreement between theory and experiment for two spheres cooperative scattering is quite good. In a future publication we will explore the enhancement phenomena in greater detail as well as study the complete Mueller matrix.

TABLE I. Comparison between theory and experiment for contacting spheres for three different sizes as a function of orientation angle. The subscripts E and T refer to experimental[2] and theoretical values respectively.

					Orientation Angle					
α	0	10	20	30	40	50	60	70	80	90
$|S(0)|_E$	15.96	14.79	13.87	13.08	13.08	13.44	14.13	14.86	15.07	14.97
$|S(0)|_T$	17.28	16.61	15.09	13.85	13.56	13.85	14.43	15.14	15.45	15.44
$\phi(0)_E$	77.5	75.2	71.7	63.6	55.6	48.5	42.8	41.5	42.5	43.0
$\phi(0)_T$	77.2	76.0	71.6	54.9	48.6	44.5	42.9	43.6	44.4	44.4
P_E	0.709	0.776	0.889	1.17	1.52	1.83	2.13	2.29	2.28	2.25
P_T	0.785	0.825	0.978	1.27	1.60	1.88	2.11	2.28	2.30	2.27
Q_E	3.20	2.94	2.69	2.42	2.22	2.07	1.97	2.02	2.09	2.10
Q_T	3.46	3.31	2.94	2.55	2.28	2.13	2.08	2.18	2.19	2.22
$|S(0)|_E$	21.97	20.70	19.56	19.86	20.59	21.86	23.70	24.60	24.68	24.74
$|S(0)|_T$	23.27	22.22	20.71	20.62	21.12	22.26	24.08	24.96	25.05	25.17
$\phi(0)_E$	109.5	103.2	90.0	77.5	67.2	59.6	56.4	55.0	55.2	55.6
$\phi(0)_T$	101.3	97.23	86.03	73.5	64.1	57.24	54.7	53.9	53.9	54.6
P_E	-1.04	-0.671	0.0	0.611	1.13	1.57	1.86	2.00	2.00	1.99
P_T	-0.649	-0.399	0.204	0.831	1.31	1.71	1.98	2.09	2.09	2.07
Q_E	2.94	2.86	2.78	2.76	2.70	2.68	2.80	2.86	2.88	2.90
Q_T	3.24	3.13	2.94	2.81	2.70	2.66	2.79	2.86	2.88	2.91
$|S(0)|_E$	27.06	28.29	31.86	33.70	36.46	39.22	41.83	43.18	43.56	43.18
$|S(0)|_T$	23.51	24.19	27.61	30.70	34.66	39.12	41.67	42.78	42.95	43.31
$\theta(0)_E$	125.7	117.8	101.1	88.3	78.6	72.6	71.1	71.6	72.0	71.4
$\theta(0)_T$	130.0	120.0	100.4	86.0	74.7	69.9	68.4	69.3	68.3	68.5
P_E	-1.44	-1.21	-0.556	0.091	0.659	1.07	1.24	1.25	1.23	1.26
P_T	-1.38	-1.07	-0.457	0.193	0.838	1.23	1.40	1.38	1.45	1.45
Q_E	2.10	2.29	2.86	3.08	3.27	3.42	3.62	3.74	3.79	3.74
Q_T	1.64	1.93	2.48	2.80	3.05	3.36	3.54	3.66	3.65	3.68

Size parameters for the three groups: $x=3.120$, $N=1.365$; $x=3.152$, $N=1.366$; $x=4.678$, $N=1.363$.

REFERENCES

1. R. T. Wang, J. M. Greenberg, and D. W. Schuerman, Opt. Lett. 6, 543
   (1981).

2. D. W. Schuerman and R. T. Wang, "Experimental Results of Multiple
   Scattering," Final Report, ARCSL-CR-81-003 (U.S. Army Chemical
   Systems Laboratory, Aberdeen Proving Ground, Md., July, 1980).

3. J. H. Bruning and Y. T. Lo, IEEE Antennas Propag. AP-19, 378-390
   (1971).

4. J. H. Bruning and Y. T. Lo, "Multiple Scattering by Spheres," Tech.
   Rept. 69-5 (Antenna Laboratory, University of Illinois, Urbana, Ill.
   (1969).

5. G. W. Kattawar and T. J. Humphreys, "Electromagnetic Scattering from
   Two Identical Pseudospheres," in Light Scattering by Irregularly
   Shaped Particles, D. W. Schuerman, ed., (Plenum, New York, 1980)
   pp. 170-190.

FIGURE CAPTIONS

Fig. 1. Extinction coefficient calculations for two spheres as a
function of distance of separation (kd). The spheres have a
size parameter x=0.9283 and a refractive index N=1.54. The
case $\alpha=0°$ corresponds to end-on incidence while $\alpha=90°$ corres-
ponds to broadside incidence. The subscript ℓ refers to the
incident beam polarized in the scattering plane while r is
for incident polarization perpendicular to the scattering
plane.

Fig. 2. (a-c) P-Q plots for the comparison between theory and experi-
ment for different sizes and refractive indices for two spheres
in contact and for orientations from end-on incidence (0°) to
broadside indicence (90°). The point labelled NIS is the P-Q
value obtained for non-interacting spheres. Curve d is for a
case of larger separation.

Fig. 3. (a-b) Scattered radiance at 90° perpendicular to the scatter-
ing plane (I_r (90°)) as a function of orientation angle α.
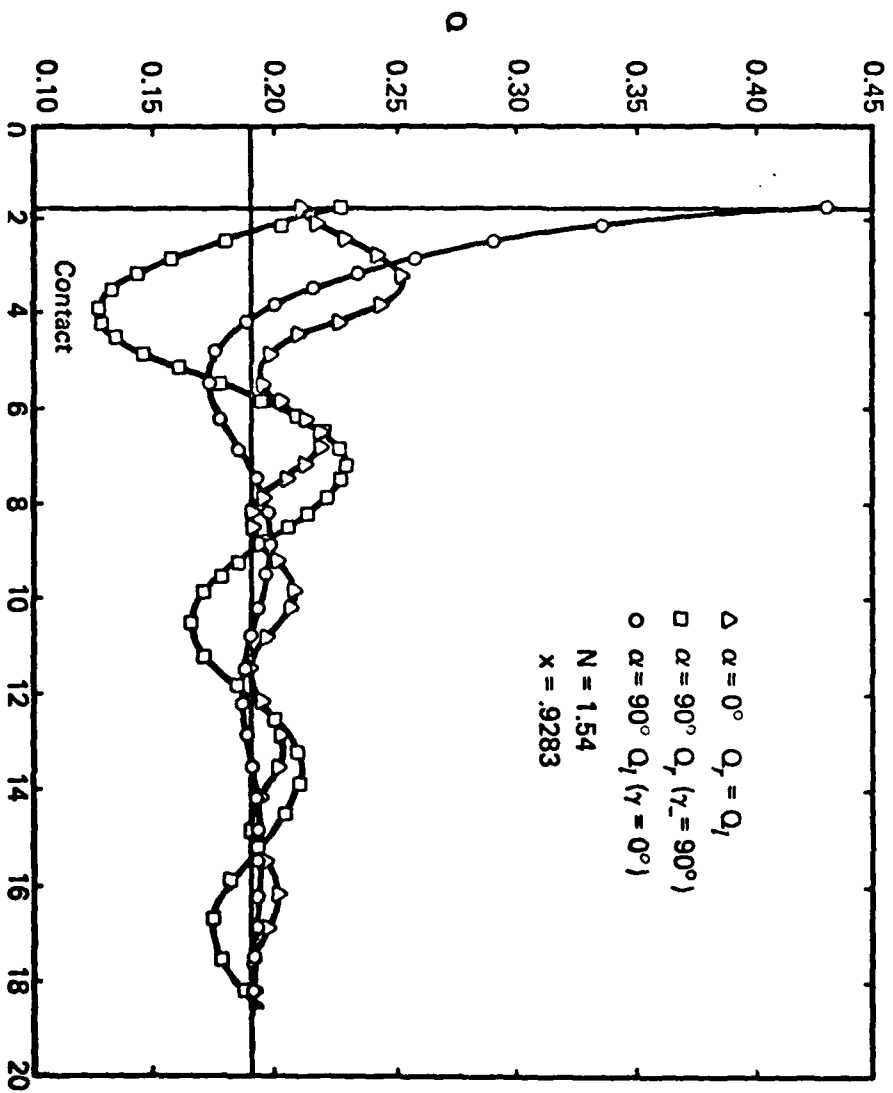Note at $\alpha=45°$ the symmetry axis of the particle bisects the
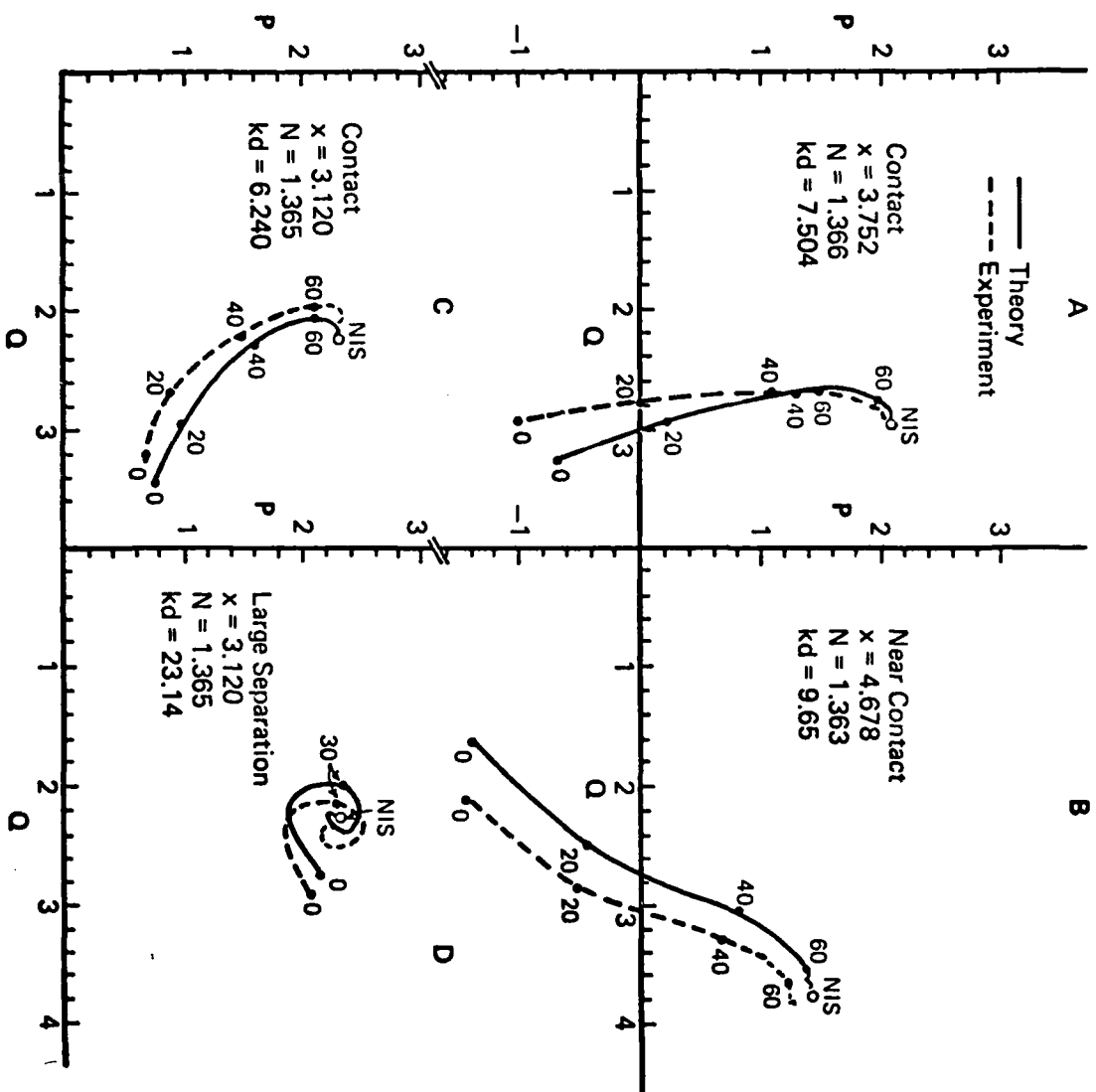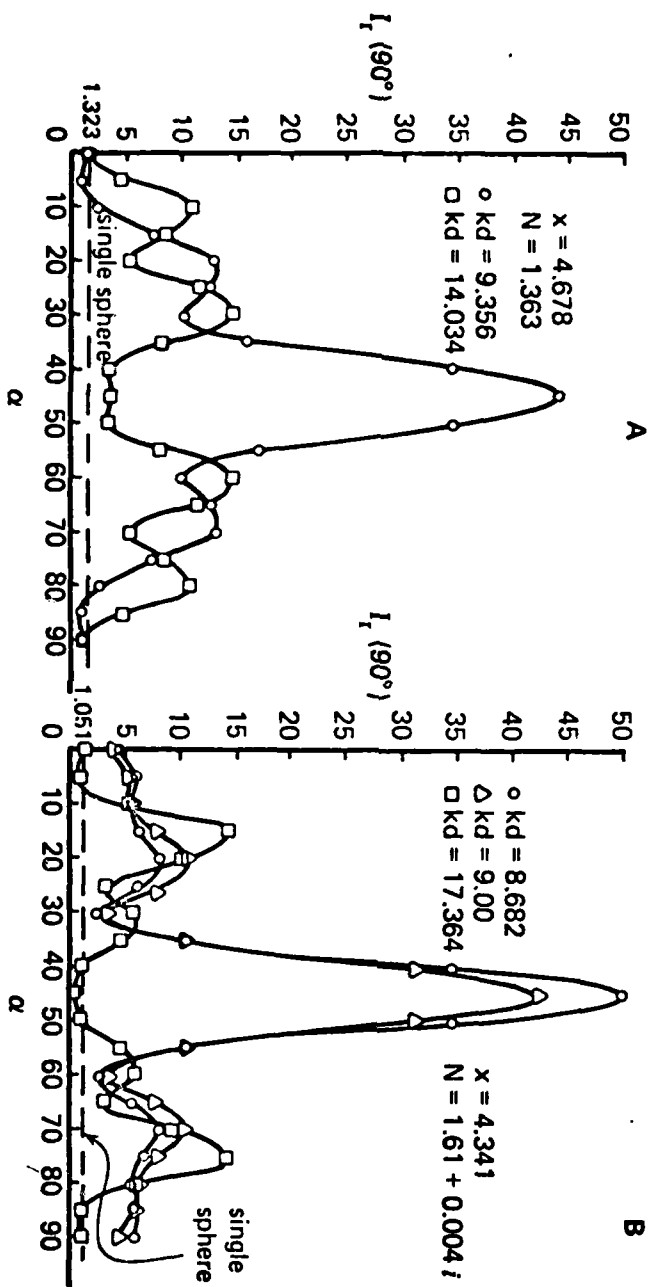scattering angle corresponding to specular reflection.

Figure 1

Figure 2

Figure 3